EUCLIDEAN DIVISION AND THE CONTINUED FRACTIONS, PERIODICITY AND IRRATIONAL NUMBERS

Authors: Maria Stratigou*, Despina Maragou, Paraskevas Politis***, Giorgos Petsaggourakis****, Vasilis Niarhakos*****, Vasilis Hondros******, Dimitris Oikonomakis[7*], Manolis Kyrios[8*], Konstantina Mountaliani[9*], Pantelis Xagoraris[10*], Antonis Metallidis[11*], Claudia Ntoubrava[12*], Nikos Melissourgakis[13*]**

Supervisor: Dr. Sophia Birba Pappa sbirba@math.uoa.gr

*mariastratigou98@gmail.com ,** despina.maragou2@gmail.com ,
*** elpolitis25@yahoo.gr ,**** boyrockguitar@hotmail.com ,
***** bniarchakos@gmail.com ,****** basilobillaros@hotmail.gr ,
7* oikojim@hotmail.gr ,9* kwnstantinabestof@hotmail.gr ,
11* antonis051999@gmail.com ,12*claudia.alexandra210@yahoo.com ,
13* nikosm2011@hotmail.gr
Zanneio Experimental Lyceum of Piraeus, Greece 18531, Kolokotroni 6 Piraeus

ABSTRACT

This paper concerns the description of rational and irrational numbers, based on the Euclidean Division and Continued Fractions and intends to highlight that, despite the fact that quadratic irrationals are irrationals, they governed by a peculiar periodicity which was known in ancient times through geometry and is readily apparent today through the form of continued fractions (Euler, Lagrange, Gauss).

The Euclidean Division (anthyphairesis) for whole numbers and magnitudes was presented by Euclid in the seventh and tenth Book of Elements respectively, which are attributed to Theaetetus, reflecting the Pythagorean tradition. The result of Euclidean Division between two whole numbers a and b (a>b) (rational number a / b) is obtained by the finite process of finding the greatest common divisor of two numbers, while the process of finding common measure between two magnitudes a and b (a>b) is either finite (rational number a / b) or infinite (irrational number a / b). When, in the process of infinite case, periodicity is appeared (self-similarity) then the parts – magnitudes of division are quadratic irrationals – a fact which separates them from the other irrationals. Therefore, the structure of quadratic irrationals resembles the one of fractals, which are governed by self-similarity. The finite case is represented by finite continued fractions while the infinite one by infinite periodic or not periodic continued fractions, where the successive quotients of Euclidean division appear in the expansion of continued fractions.

Quadratic irrationals find frequent use in science and art.